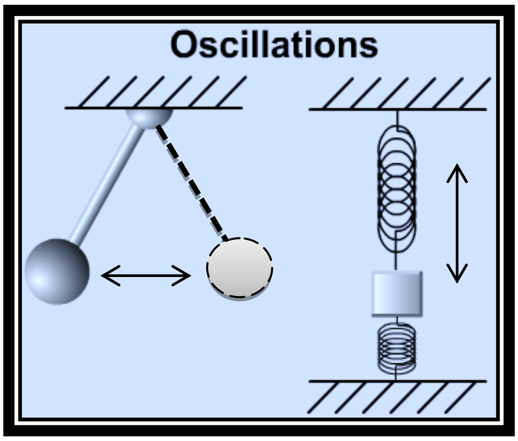
British Middle East Center
for Studies and Research

Wheel and Axle



Author: Joseph Harrison

Research Ans Aerospace Engineering Expert,
Monash University

Introduction

A solid wheel and axle is held at the top of an incline, released and allowed to roll down the incline for a given distance. You are to use an energy method to determine the radius of gyration for the solid body. As part of your investigation you need to examine the principles of angular motion and carry out a practical element relating to the following:

- Take part in the assignment launch and conduct a practical investigation.
- The energy contained within the disc at the starting point compared with the energy at the end of the run.
- The relationship between distance travelled and linear and angular motion.
- The effects of energy transfer.
- Consideration of factors that may be involved but not mentioned here.

First the experiment is carried out to determine the average time as well as the distance is calculated with the height and the diameters of the wheel. The way this was done was that a team had to carry out the experiment and not an individual. The reason for this is because as the wheel roles down the required distance someone has to keep track of time while the other has to reset the track again. The experiment had to be carried out three times in order to work out the average time because each time the experiment was carried out a different reading emerged which was caused by human error.

To get more accurate results or even to recheck the result we can by redoing the test to ensure that we've obtained the equivalent results. Or we can increase the distance of the slope, or increase the mass to compare their radius of gyration. So to conclude the experiments purpose was to gain that data (Time, Distance, Weight, Radius, Diameter, and Height) and therefore use that data to go through a method to find the radius of gyration and this includes the following calculation below.

$t_{Average} = 12.5\ seconds$

$Length = 1.25m$

$Small\ diameter = 0.012m$

$Height = 0.14m$

$Small\ radius\ (r) = 0.006m$

$Big\ diameter = 0.15m$

$Big\ radius\ (R) = 0.075m$

Volume of the Disc:

$V_{Disc} = \pi 0.075^2 \times 0.022 = 3.887 \times 10^{-4} cm^3$ However the volume of the section that will need to be subtracted from the total of the Volume of both the Disc and the Axial.

$V_{Clearance Hole} = \pi(6.25 \times 10^{-3})^2 \times 0.022 = 2.699 \times 10^{-6}\text{cm}^3$

Now we need to determine the volume of the Axial

$V_{Axial} = \pi(6.25 \times 10^{-3})^2 \times 0.12 = 1.472 \times 10^{-5}\text{cm}^3$

Now we need to determine Total volume

$V_{Total} = (V_{Disc} + V_{Axial}) - V_{Clearance Hole}$

$V_{Total} = (3.887 \times 10^{-4} + 1.472 \times 10^{-5}) - 2.699 \times 10^{-6}$

$= 4.00 \times 10^{-4}\ \text{cm}^3$

We can now determine the mass, by rearranging the following equation of density (ρ) for its mass:

$\rho = \frac{m}{V}$ $\therefore Mass = density\ x\ volume$

$Density\ = 8000\ kg/m^3$

$Mass = 8000\ x\ 4\ x\ 10^{-4}$

$Mass = \ 3.2\ kg/m^3$

Now we workout the maximum velocity:

$$V = \frac{Distance}{Time}$$

$$Vmax\ = \frac{2(1.265)}{12.39}\ = 0.2042\ m/s$$

We can now work out the value of its angular velocity (the speed of the disc):

$V = \omega r$

$$\therefore \omega = \frac{V}{r}$$

$$= \frac{0.2}{6.25 \times 10^{-3}}$$

$\omega\ =\ 33.3\ rads/s$

P.E and K.E (Linear and Rotational)

As the disc is at the top of the slope, the disc is containing Potential Energy (P.E), when the disc is in motion and no longer stationary, travelling down the slope, the disc then converts its Potential Energy in to 2 forms of Kinetic Energy (K.E), these two types of Kinetic that was formed from the conversion of the Potential Energy, as it travelled down its given distance, are identified as Linear and Rotational Kinetic Energy.

$Pe = mgh$

$Pe = 3.2 \times 9.81 \times 0.14$

∴ = 4.39 Joules

$KeLin = \frac{1}{2} mv^2$

$Kerot = ½ I\omega^2$

$mgh = ½ mv^2 + ½ I\omega^2$

$m =$ 3.2kg

g = 9.81 m/s^2

V = 0.2 m/s

ω = 32 rad/s

I Is the mass moment of inertia of the Disc, which we need to determine.

$I = mk^2$

And, which can be used as a checking method.

$I = \frac{mR^2}{2}$

k is the radius of gyration, where a single radius at which the total mass of the body may be assumed to be concentrated, we need to determine this value in order to obtain the value of Inertia, which in result will help us determine the Energy Equation.

$mgh = ½ mv^2 + ½ mk^2\omega^2$

Multiplying the $\frac{1}{2}$ with mV^2 and $(mk^2)\omega^2$ gives:

$mgh = \frac{mV^2}{2} + \frac{mk^2\omega^2}{2}$

Because m is multiplied for both variables, we can rewrite the equation as $mgh = \frac{m(V^2+k^2\omega^2)}{2}$

Divide both sides by m gives $gh = \frac{(V^2+k^2\omega^2)}{2}$ and then multiply 2 by both sides give

$2gh = V^2 + k^2\omega^2$

Subtract V^2 both side gives

$2gh - V^2 = k^2\omega^2$ Then divide both sides by ω^2 gives $\frac{2gh-V^2}{\omega^2} = k^2$

Now square root both side gives: $k = \sqrt{\frac{2gh-V^2}{\omega^2}}$

$$k^2 = \sqrt{\frac{2gh - v^2}{\omega^2}}$$

$$k^2 = \sqrt{\frac{2gh - 0.2^2}{33.3^2}}$$

$k = 0.049\, m$

$h = height$

$I = moment\, of\ inertia$

$\omega = angular\, velocity$

$k = radius\ of\ gyration$

If we go back to our Inertia equations, both can be expressed as the following

$$I = mk^2 = \frac{mR^2}{2}$$

If we make (k) the subject we can use this as a checking device to see if our value is correct. So dividing the mass on both sides gives us:$mk^2 = \frac{mR^2}{2}$

$$k^2 = \frac{R^2}{2}$$

Square root both side gives: $k = \sqrt{\frac{R^2}{2}} =$ $k = \sqrt{\frac{0.075^2}{2}}$

Which gives us a value of 0.05 m, which is near enough to our value of$k$, where our value was 0.049 m

Now knowing all the values of each variable we can determine its Inertia of the Disc

$I = 3.2 \times 0.049^2$

∴ = 0.0032

Now we can determine our value for its

Rotational K.E $= \frac{1}{2} I\omega^2$

$$\frac{1}{2} \times 0.0032 \times 33.3^2$$

∴ = 4.25 Joules

Linear K.E$= \frac{1}{2} mV^2$

$$\frac{1}{2} \times 3.2 \times 0.2^2$$

∴ = 0.064 Joules

Therefore to conclude the results demonstrate that the checking was correct and the answer that was gained was close to the actual result gained for the radius of gyration in task 1. For the radius of gyration for task one the result was 0.049 and after checking the result the answer was 0.05, therefore you can see the similarity of the results when they are compared.

Task 2

1. A helical spring has a natural frequency of vibration of 2Hz when it supports a mass of 4kg. The body is given a free vibration of amplitude 5mm. Determine:

    a. The period of the motion.
    b. Stiffness of the spring
    c. The displacement after it has passed its equilibrium position by 0.02s in an upward direction.
    d. The velocity and acceleration at this point

2. In a simple slider - crank mechanism the crank is 50 mm long and the connecting rod is 200 mm long, the crank rotates at15 rev/s. Find;

    a. The velocity and acceleration of the piston when the crank is 30° from top dead centre.
    b. The maximum velocity and acceleration of the piston.

3. Explain, with the aid of diagrams, the effect of resonance related to a spring mass system and how this would affect the "ride" when related to a motor vehicle suspension system?

$Mass = 4kg$

$Frequency = 2$

$T = \frac{1}{F}$

$Period\ of\ motion,$

$Time = ½\ s = 0.5$ seconds

Stiffness of the spring:

$$T = 2\pi \sqrt{\left(\frac{m}{k}\right)}$$

$$K = \frac{m}{(\frac{T}{2\pi})^2}$$

$$K = \frac{4}{\left(\frac{0.5}{2\pi}\right)^2}$$

$K = 631.65\ N/m$

Displacement after it has passed its equilibrium position by 0.02s in an upward direction. Using the following equation to determine how much the spring moved by 0.02 seconds as it passes its equilibrium point.

$c. \omega = 2\pi f$

$\omega = 12.566$

$t = 0.02$

$\theta = \omega t$

$\theta = 12.566 \times 0.02 = 0.2513\ rads - 1\ or\ 14.4°$

Determining the total theta

$\theta_{Total} = 14.37 + 90 = 104.37°$

$r = 5\ mm$

$x\ =\ r\ cos(\omega t)$

$x\ =\ r\ cos(\theta)$

$The\ displacement\ after\ 0.02s$

$x =\ 5\ cos\ (14.4\ +\ 90)\ =\ -1.24mm$

<u>The maximum acceleration.</u>

$x\ =\ r\ cos(\omega t)$

$\omega t\ =\ 104.4$

$v\ =\ -r\omega sin(\omega t)$

$a\ =\ -\omega^2 t$

$r\ =\ 5$

$\omega\ =\ 12.566$

$t =\ 0.02$

$x\ =\ -1.24mm$

<u>The maximum velocity.</u>

$v\ =\ -5\ \times\ 12.566\ sin\ (104.4)$

$v\ =\ -60.856\ mm/s$

$v\ =\ -0.061\ m/s^2$

$a\ =\ -(12.566)^2\ \times\ 0.02$

$a = 195.8\ mm/s^2$

$a = 0.1958\ m/s^2$

In a simple slider - crank mechanism the crank is 50 mm long and the connecting rod is 200 mm long, the crank rotates at15 rev/s. Find;

<u>The velocity and acceleration of the piston when the crank is 30° from top dead centre.</u>

$Crank = 50\ mm$ $\quad$ $Rotation = 15\ rev/s$

$Rod = 200mm$ $\quad$ $Angle\ from\ top = 30°$

Using the following equation to determine the Velocity of the Piston

$$v = -\omega r \left[\sin\theta + \left(\frac{r}{2L}\right)\sin 2\theta\right]$$

$$v = -94.25 \times 50 \left[\sin 30 + \left(\frac{50}{2 \times 200}\right) \sin(2 \times 30)\right]$$

$$v = -2866.4\ mm/s$$

$$a = -\omega^2 r\,[\cos(\theta) + \left(\frac{r}{L}\right)\cos 2\theta]$$

$$a = -94.252^2 \times 50 \left[\cos 30 + \left(\frac{50}{200}\right)\cos 60\right]$$

$$a = -440167.03\ mm/s^2$$

<u>The maximum velocity and acceleration of the piston.</u>

Using the following equation to determine the Maximum Velocity, by making v the subject

$$v = -\omega r \left[\sin\theta + \left(\frac{r}{2L}\right)\sin 2\theta\right]$$

$$v = -94.25 \times 50 \left[\sin 90 + \left(\frac{50}{2 \times 200}\right) \sin(2 \times 90)\right]$$

$$v = -4712.5\ mm/s$$

$$a = -\omega^2 r \left[\cos(\theta) + \left(\frac{r}{L}\right)\cos 2\theta\right]$$

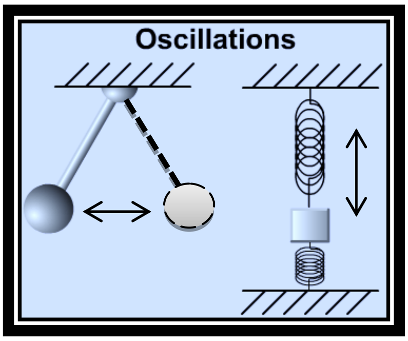
$$a = -94.252^2 \times 50 \left[\cos 0 + \left(\frac{50}{200}\right)\cos 0\right]$$

$$a = -555191\ mm/s^2$$

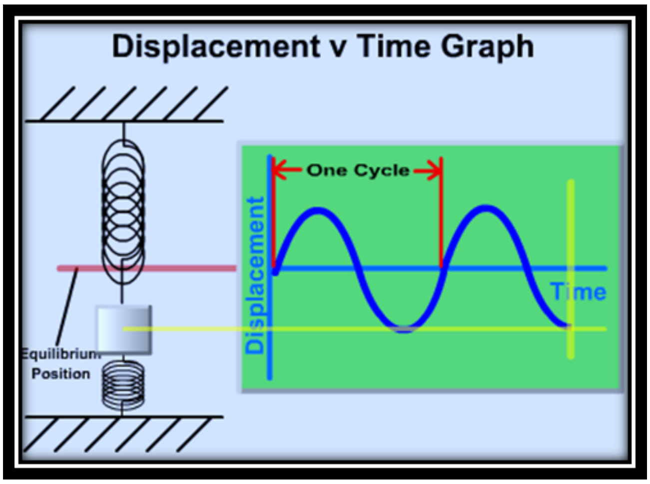
To conclude the maximum acceleration as well as the maximum velocity was checked using sine wave to see if the peak of the wave was similar to the results gained here and therefore it was.

Task 3

When we look at for e.g. the movement of an object back and forth over a fixed range of positions, such as the movement of a swinging ball attached with a cable, or the bouncing up and down of a weight on the end of a spring. As you can see in the diagram provided here we can see that the The weight and the mass-spring system are both oscillating. However to describe their oscillation we will use a diagram to demonstrate the concept.



The diagram shows how displacement describes the distance of the object from its equilibrium position, also direction. Displacement can be negative as well as positive. The displacement amplitude tells us how 'big' the oscillations are - we can use the peak value (the maximum positive displacement from the equilibrium position) or the peak-to-peak value (the distance between negative-maximum to positive-maximum). The time period of the oscillation is the time taken for the object to travel through one complete cycle. We can measure from the equilibrium position, as in the animation, or from any other point on the cycle, as long as we measure the time taken to return to the same point with the same direction of travel.

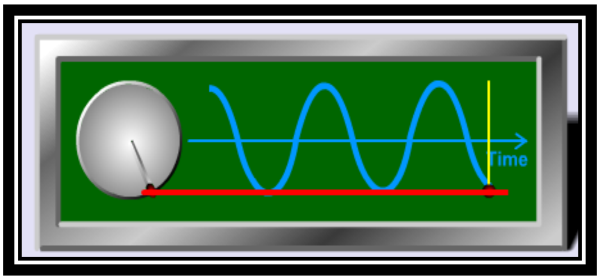


When our oscillating object reaches maximum displacement (when it is as far from equilibrium as it can get) it changes direction. This must mean that there is an instant in time when it is not moving and so at the point of maximum displacement, its speed is zero. Then it speeds up in the opposite direction, and travels fast through the equilibrium position before starting to slow again in preparation for the next change in direction. So when velocity is at maximum, displacement is zero, and when displacement is maximum, velocity is zero. The period of oscillation is the same, but the plots for displacement and velocity do not line-up in time, one is shifted along compared to the other. This shift along in time is called a phase angle. (Salford 2012)

If we use an swing in a playground as an example, the swing is therfore first pulled back, and then pushed forward. It then oscillates back and forth 'on its own' until it slowly comes to rest at its equilibrium position, the middle position between the two extremes of displacement. This is usually the place where an object will naturally rest if no external forces are applied to it.

Once the swing has been pulled away from its equilibrium position, the force of gravity will act to bring it back. This force will always act in a direction towards the equilibrium position, and is known as a restoring force. If the swing is pulled higher into the air (larger displacement amplitude) then there will be a larger restoring force acting on it, and when the pusher lets go it will travel further. This shows that the restoring force is proportional to the distance of the swing from the equilibrium position.

An increase in mass increases the inertia (reluctance to change velocity i.e. reluctance to accelerate / decelerate) of the object. In this example we have not used gravity, so the heavy mass does not 'sag' on the springs. massive objects have inertia even in outer space where they have no weight. An increased inertia means that the springs will not be able to make the mass change direction as quickly. This increases the time period of the oscillation. The greater the inertia of an oscillating object the greater the time period; this lowers the frequency of its oscillations. The oscillating objects we've looked at at to now are all vibrate with a rather special 'shape' or waveform. This waveform is



If this displacement is plotted on a graph with the y axis showing displacement and the x axis time the wave shape is revealed. In the case a disc spinning at a constant speed a sin wave is created.

called sine wave. (Salford 2012)

The equations used below underline what we've been talking about. An oscillation follows simple harmonic motion if it fulfils the following two rules:

- Acceleration is always in the opposite direction to the displacement from the equilibrium position.
- Acceleration is proportional to the displacement from the equilibrium position.

The acceleration and displacement are linked by the following equation:

Acceleration = $- \omega^2 * x$ Here ω is called the angular frequency of oscillation, and is given by $2\pi / T$ or $2\pi f$ Where T is the period of oscillation (s), f=1/T = frequency of oscillation (Hz) and x is the displacement (m). Using Newton's 2nd Law (F=ma) we can show that the Force on the object due to inertia will be:

$$F = -m \omega^2 x$$

Using Hooke's Law for springs(F=-kx), we know that the force on the oscillating mass due to the springs is simply F=-kx

These two forces are always in balance, so

$$m \omega^2 x - kx = 0$$

From this we can find the resonant frequency:

$$\omega^2 = (kx) / (mx) = k / m,$$

$$\omega = \text{sq.root } (k / m)$$

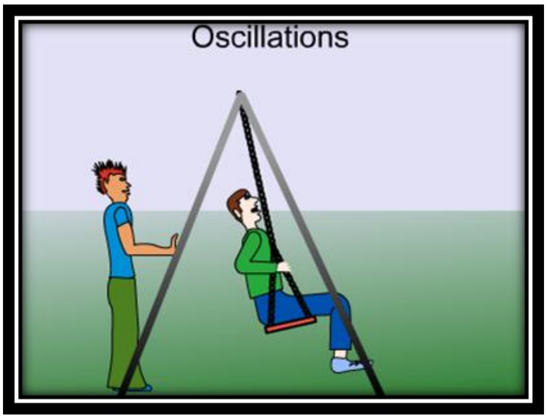
$$f = 1/ 2\pi * \{ \text{sq.root } (k / m) \}$$

damping is a loss of energy from, in that example, movement (kinetic) energy to heat. There other examples of damping used but in this case we used the playgound swings.

Free Oscillations, Forced Oscillations and Resonance

If an oscillator is displaced and then released it will begin to vibrate. If no more external forces are applied to the system it is a free oscillator. If a force is continually or repeatedly applied to keep the oscillation going, it is a forced oscillator.

If the swing is pushed just once it acts as a free oscillator and the damping effects of air resistance and losses at the pivots mean it will eventually stop swinging. If the swing is pushed each time it reaches a certain point it behaves as a forced oscillator and will continue to swing for as long as energy is supplied.

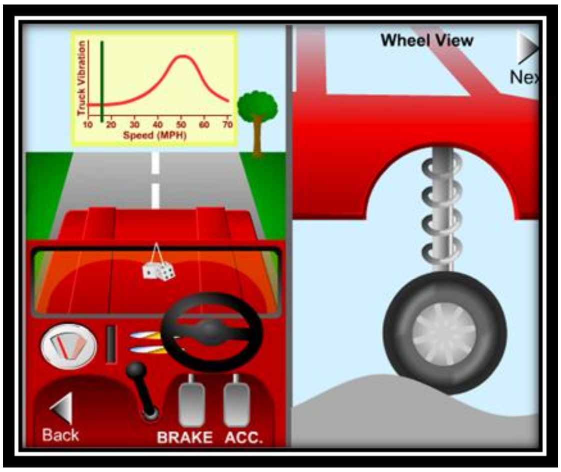


The 'pusher' normally pushes the swing every time it reaches its maximum negative displacement. If the swing is simply lifted and let go it will swing with a natural frequency. This frequency is determined by a number of factors, of which the most important is the length of the swing from pivot to seat. If the swing is pushed with a long interval between pushes, the swing will not recieve enough energy to replace that lost by damping. Also, the pushes may be 'out of synch' with the swing's natural period of oscillation. This might mean that sometimes the pusher would be pushing when the swing is in the wrong location. If the person pushes with a shorter interval between pushes, he may again be 'out of synch' with the natural frequency of the swing. (Salford 2012)

The natural frequency that the swing 'wants to' oscillate at is called its resonant frequency. If the pusher pushes at the swing's resonant frequency, the amplitude of oscillation will build up. With each period, the pusher will add more energy to the system. Eventually, what usually happens is that energy supply equals energy loss (to damping), and the amplitude stabilises at some large value.

Using a car as an example. At resonance the amount of energy lost due to damping is equal to the rate of energy supply from the driver. The driver is the source of external energy that keeps the oscillations going for e.g., the person pushing a kid on a swing. Increasing the damping will reduce the size (amplitude) of the oscillations at resonance, but the amount of damping has next to no effect at all on the resonant frequency.

Damping also has an effect on the 'sharpness' of a resonance; sharpness is a not-very scientific way of describing how sensitively the resonance is tuned, and is sometimes called the 'Q-factor' by engineers. If damping is very small, a system will only oscillate a little if driven even slightly above or below the 'right' frequency - but when the driver hits the resonant frequency 'bang-on', suddenly the oscillations can get very large. Conversely, if damping is large, the amplitude of oscillations at resonance will decrease, but if the driver shakes (excites) the system at the 'wrong' frequency, the system will still respond quite strongly. This means there will be less of a resonant effect, but that it will happen over a larger range of frequencies. (Salford 2012)



The diagram below shows how the suspension of a car can be used to demonstrate resonance and damping. From the diagram below we can see that if the speed of the car is low then the vibration wil allso be low but as the speed in creases so does the vibration cuased by the surface of the road. However to a certain heigh speed the vibration will reach its peak and therfore after that the vibration will become stable agin eventhough the speed is still increasing.

Conclusion

To conclude, the results that were taken from the practical work that was carried out in order to calculate the time taken for the disk to travel along the distance given. The result show the correct radius of gyration which therefore means the experiment was carried out correctly. The aim of this goal was obtaining all the data from the practical experiment. We wanted to find the radius of gyration; this was the main objective, we worked in groups as each member was given a task, these tasks consisted of taking the time for the disk to travel, as well as calculating the distance and the height. Also calculating the weight and checking the material of the disk to find out the mass of the material which was steel. Therefore the level of communications was high because we had to continuously communicate to achieve the results. After the result where gathered the finding of the radius of gyration was simple as it followed steps and these include finding the total volume first then use the results for the volume to determine the mass, then maximum velocity was determined in order to work out the angular velocity. Then the radius of the gyration was determined by rearranging the energy equation. Therefore if we compare the result they are pretty close to each other the reason why they are not exactly the same is because of human error and these include rounding up numbers before the final answer.

Task	Weeks						
	1	2	3	4	5	6	7
Analyse tasks							
Take relevent notes							
Understand the the behaviour of dynamic mechanical systems							
Dynamic mechanical systems							
Dynamic mechanical systems							
Constructing Gantt charts							
Understand the effects of energy transfer in mechanical systems							
Understand [illegible]							
Attampt all taskes							
Finished and ready to be hand-in							